

String Gas Cosmology after Planck

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Abstract.

We review the status of String Gas Cosmology after the 2015 Planck data release. String gas cosmology predicts an almost scale-invariant spectrum of cosmological perturbations with a slight red tilt, like the simplest inflationary models. It also predicts a scale-invariant spectrum of gravitational waves with a slight blue tilt, unlike inflationary models which predict a red tilt of the gravitational wave spectrum. String gas cosmology yields two consistency relations which determine the tensor to scalar ratio and the slope of the gravitational wave spectrum given the amplitude and tilt of the scalar spectrum. We show that these consistency relations are in good agreement with the Planck data. We discuss future observations which will be able to differentiate between the predictions of inflation and those of string gas cosmology.

1. Introduction

The recently released Planck results [1] have further confirmed the predictions of the six parameter Λ CDM cosmological model. Even though these six parameters describe properties of the current universe, their values are quite mysterious without going back to processes which happened in the very early universe. In particular, the origin of the almost scale-invariant spectrum of almost adiabatic curvature fluctuations with a slight red tilt which has now been confirmed by Planck and many other experiments has no explanation in the context of late time cosmology.

Inflationary cosmology [2] is the current paradigm for explaining the overall homogeneity and spatial flatness of space and the origin of the observed spectrum of fluctuations [3]. The inflationary scenario is based on the potential energy of some scalar field yielding a period of (almost) exponential expansion of space. In simple models of inflation, however, the energy scale at which inflation takes place is of the order of 10^{16} GeV, much closer to the Planck scale than to scales which have been explored in particle physics experiments, and in fact even closer to the string scale [4]. Hence, in order to truly understand the mechanism of inflation, it appears to be necessary to tackle superstring cosmology.

Another reason for turning to string theory as a framework for inflationary cosmology is the “ η problem” (see e.g. Section 27.2 in [5]). In order not to produce a too large amplitude of fluctuations, the scalar field potential must be sufficiently flat. For simple scalar field potentials, the slow-roll inflationary dynamics takes place at field values larger than the Planck mass m_{pl} . This is also a field range for which the slow-roll inflationary trajectory is a local attractor in initial condition space [6], even including linear metric fluctuations [7] \ddagger . In order to understand physics in this field range, it seems necessary to embed inflation into an ultraviolet complete theory of quantum gravity such as superstring theory.

In most current approaches to superstring cosmology, the theory is treated in an effective field theory limit in which scalar fields motivated by string physics are coupled to Einstein gravity or dilaton gravity (see e.g. [12] for recent reviews on inflation in this framework). However, this approach misses one of the key symmetries of string theory, namely the “T-duality” symmetry [13]. To illustrate one manifestation of this symmetry, assume that space is toroidal. In this case, the perturbative spectrum of string states contains both string momentum and winding modes. The energy of the string momentum modes scales as $1/R$, where R is the radius of the torus, whereas the energy in the winding modes increases linearly in R . Thus, the spectrum of string states is unchanged under the transformation $R \rightarrow 1/R$ (in string units). The string vertex operators are consistent with this symmetry, and if we postulate that the symmetry extends to the non-perturbative level, we predict the existence of D-branes [13, 14].

Another feature of superstring theory which is missed in the effective field theory description is the existence of an infinite tower of string oscillatory modes which leads to a maximal temperature for a gas of closed strings in thermal equilibrium, the *Hagedorn temperature* [15]. It is self-evident that both the T-duality symmetry of string theory and the existence of a limiting temperature should play an important role in superstring cosmology. The *string gas cosmology* (SGC) [16] scenario which will be reviewed here is an approach to superstring cosmology which is based on the T-duality symmetry of string theory and string thermodynamics [17]. As was realized much later [18], string gas cosmology yields an alternative to inflation for explaining the origin of the inhomogeneities and anisotropies which are now mapped out by cosmological experiments such as the Planck satellite mission.

Some readers may be under the impression that observations have proven that there was a period of inflation in the early universe. However, all that the observations show is that there is some mechanism which produces an almost scale-invariant spectrum of nearly adiabatic fluctuations with a small red tilt on scales which were larger than the Standard Big Bang horizon at early times (times comparable to the time t_{rec} of last scattering). That such a spectrum yields acoustic oscillations in the angular power spectrum of the cosmic microwave background (CMB) and baryon acoustic oscillations in the galaxy power spectrum was realized [19] more than a decade before

\ddagger However, beyond linear order the story may be very different [8, 9, 10]. Also, in the case of small field inflation there is an initial condition problem [11].

the development on the inflationary scenario. It is true that cosmological inflation is the first model where such a spectrum emerges from first principles, but it is not the only one (see e.g. [20] for a recent review of alternatives).

Alternatives include the *Pre-Big-Bang* and *Ekpyrotic* scenarios [22] (in which a light entropic mode obtains a scale-invariant spectrum during the contracting phase [23]), the *conformal universe* [24] or *pseudo-conformal universe* [25] (in which a scale-invariant spectrum is generated from an emergent phase by a moving Galileon field which induces squeezing of the curvature fluctuations), the *matter bounce* scenario [26, 27] in which a scale-invariant spectrum of curvature perturbations is generated from initial vacuum fluctuations on scales which exit the Hubble radius during the matter-dominated phase of contraction, and *string gas cosmology*, the topic of this review. In the same way that inflation may find an embedding within superstring theory, it is also possible that one of the alternatives to inflation ends up being realized in string theory. For example, the Ekpyrotic scenario was initially motivated from a stringy construction, heterotic M-theory [28], and there is a recent realization [29] of the matter-bounce scenario in which an S-brane originating from extra states becoming massless at a T-dual point mediates the transition from contraction to expansion. String gas cosmology is, obviously, from the outset firmly rooted in superstring theory.

In the following we first present a review of string gas cosmology (see e.g [30] for more detailed reviews). We then confront the predictions of string gas cosmology with recent observations, and we close this article with a discussion of some major challenges facing the scenario.

2. String Gas Cosmology

Standard Big Bang Cosmology is based on coupling a gas of point particles to a background space-time. “String Gas Cosmology” [16] (see also [31]) is a modest extension of this setup which maintains the key new symmetries (T-duality) and new degrees of freedom (string oscillatory and winding modes) which distinguish string theory from point particle theories. This is achieved by replacing the gas of point particles by a gas of strings. To be specific, we consider a theory of closed superstrings, and we assume that the background space is toroidal (extensions to toroidal orbifolds are considered in [32]). We also assume that the string coupling constant is small such that the strings are the light degrees of freedom (compared to branes - for an analysis of the role of branes in string gas cosmology see [33]).

Following what is done in Standard Big Bang Cosmology, we assume that matter is in thermal equilibrium. The infinite tower of string oscillatory modes then leads immediately to a crucial difference between point particle cosmology and string cosmology: the temperature of the gas of strings has a maximal temperature T_H , the “Hagedorn temperature” [15] which is given by the string scale. Let us start with a large dilute box of strings at low temperatures. In this case almost all of the energy is in the momentum modes whose energy increases as the box contracts. This leads to a

rising temperature, as in particle cosmology. However, once the temperature approaches T_H , the energy density is high enough to excite the string oscillatory modes. Further contraction of the box will lead to a growing tower of oscillatory modes being excited at approximately constant temperature. Once the box size decreases below the string scale, the energy of the gas of strings will drift into the winding modes which become less energetic as the universe contracts, leading to a decreasing temperature. Figure 1 [16] shows how the temperature of a gas of heterotic superstrings in a box of radius R varies as the radius changes. The vertical axis is temperature, the horizontal axis the radius of the box in units of the string length. The two different curves corresponding to different total amounts of entropy - the larger the entropy the more extended the *Hagedorn phase*, the phase where T is close to T_H .

To obtain a cosmological scenario we need not only kinematics of string gas cosmology, but also dynamics. At the present time we do not have a first principles dynamics which comes from string theory. Neither Einstein gravity nor dilaton gravity can be applied in the Hagedorn phase since these frameworks are not consistent with the symmetries of string theory which are expected to be present in the Hagedorn phase. There are two possible dynamical scenarios. In the first, the universe starts in a quasi-static Hagedorn phase with all spatial dimensions wrapped by strings. The decay of string winding modes into loops triggers the dynamical breaking of the T-duality symmetry of the string gas and the transition to the radiation stage of Standard Big Bang cosmology. Figure 2 is a sketch of the time evolution of the cosmological scale factor $a(t)$ according to this scenario [16].

An alternative scenario is that the universe undergoes a cosmological bounce in which R starts out at the far left of Figure 1, i.e. with $R \ll l_s$, where l_s is the string length, passes through the Hagedorn phase, and then enters the usual radiation phase of Standard Cosmology. In terms of the light variables, the phase with $R \ll l_s$ and R increasing corresponds to a contracting phase. Thus, this second scenario corresponds to a cosmological bounce mediated by a gas of strings [34]. However, in the following we will explore the first possibility, namely that the universe starts in a long quasi-static Hagedorn phase.

String theory is mathematically well-defined in ten space-time dimensions. The idea of string gas cosmology is to start in a thermal state in which all spatial dimensions are equivalent, and to then explain why only three spatial dimensions effectively decompactify [16], as opposed to the usual approach in string motivated field theory cosmology where one assumes from the outset that our three spatial dimensions are special and one then requires some ad-hoc compactification mechanism. Thus, we start with a thermal gas of strings in which the momentum and winding modes about all spatial dimensions are excited. The presence of winding modes prevents spatial sections from expanding. The only way that a spatial dimension can become large is if the winding modes about that dimension can decay. Decay of winding modes, however, requires winding mode interactions. Since such interactions require the string world sheets to intersect, the interaction probability is negligible in more than three spatial

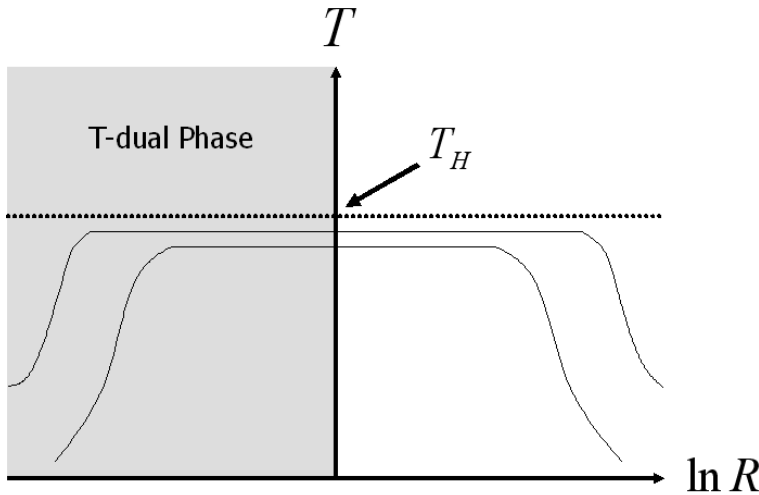


Figure 1. Temperature (vertical axis) of a gas of heterotic superstrings as a function of box radius (in units of the string length). T_H is the Hagedorn temperature, the maximal temperature which the gas of string can achieve. How close to T_H the temperature T actually gets depends on the total entropy of the system. Shown are two curves corresponding to different entropies (the top curve has a higher entropy).

dimensions, as long as there are no long-range forces between the strings [16] (see also [35] for a numerical study). The three dimensions in which winding modes can annihilate may not all start to expand at exactly the same time, but, as long as some string winding modes are still present there is an isotropization mechanism which is at work [36].

In the string gas cosmology setup the six spatial dimensions in which the winding modes were not able to annihilate are confined by the gas of winding and momentum modes to remain at the string scale [37, 38]. Thus, the size moduli of string theory are naturally stabilized in string gas cosmology. This corresponds to moduli stabilization at enhanced symmetry points [39, 40]. In the case of heterotic superstring theory it can be shown explicitly [41] that this moduli trapping mechanism is consistent with late time cosmology. The presence of winding modes can also trap shape moduli of the

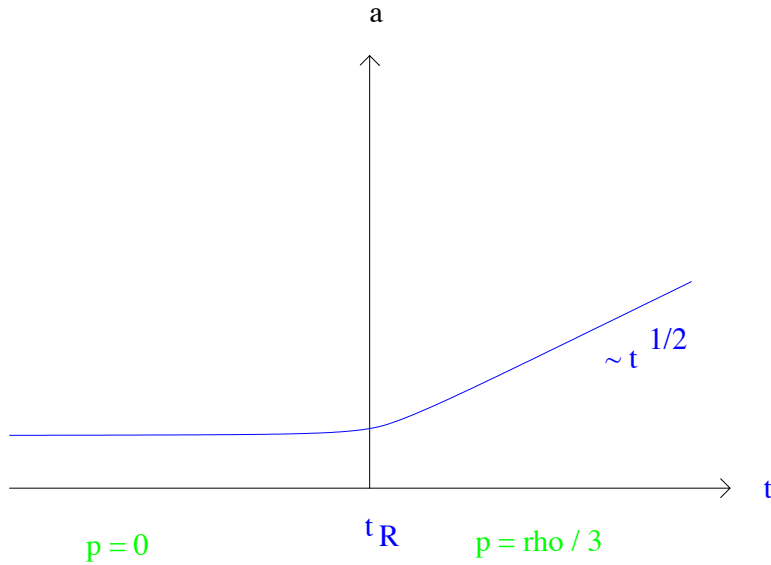


Figure 2. Evolution of the scale factor in string gas cosmology. For $t < t_R$ we are in the Hagedorn phase. At t_R a phase transition to the radiation phase of Standard Cosmology takes place.

internal dimensions, as was shown in [42]. The one modulus which is not stabilized by intrinsic stringy effects is the dilaton. The dilaton, however, can be stabilized by invoking gaugino condensation [43], without destabilizing the size moduli. Gaugino condensation then leads to (typically high scale) supersymmetry breaking [44]. The bottom line is that moduli stabilization, the Achilles heel of many other approaches to string cosmology, appears to be in good control in the context of string gas cosmology.

3. String Gas Cosmology, Structure Formation and the Planck Results

In inflationary cosmology and some of its alternatives, the source of cosmological perturbations is quantum vacuum fluctuations [3]. A justification of this idea is as follows: the exponential expansion of space dilutes the density of any excitations which

may have been present at the beginning of the inflationary phase, leaving behind a vacuum state of matter. In contrast, in string gas cosmology the initial state is a hot gas of strings in thermal equilibrium at a temperature close to T_H , the Hagedorn temperature. Hence, in string gas cosmology the source of inhomogeneities is thermal fluctuations of a gas of strings. As realized in [18], this leads to an almost scale-invariant spectrum of curvature perturbations at late times, and similarly [45] to an almost scale-invariant spectrum of gravitational waves. The spectrum of cosmological perturbations has a small red tilt, like in the case of inflation. However, the spectrum of gravitational waves has a small blue tilt [45], unlike in the case of inflation where a red tilt is inevitable (provided matter is used which obeys the usual energy conditions).

Cosmological fluctuations should be viewed as a superposition of small amplitude plane wave inhomogeneities. If we expand the full equations of motion for space-time and matter to linear order in these fluctuations, each wave will evolve independently. In inflationary cosmology, each wave corresponds to a harmonic oscillator which begins in its vacuum state on sub-Hubble scales and is stretched by the accelerated expansion of space to super-Hubble lengths where the wave function is squeezed and classicalizes via the intrinsic nonlinearities of the system [46, 47]. We now compare this setup to what happens in string gas cosmology.

A space-time diagram of string gas cosmology is shown in Fig. 3. The vertical axis is time, the horizontal axis indicates physical spatial length. The Hagedorn phase corresponds to times earlier than t_R . During the Hagedorn phase space is static and hence the Hubble radius is infinite. After the decay of the string winding modes our three dimensional space starts to expand according to the usual laws of Standard Big Bang cosmology. The Hubble radius drops to a microscopic value at t_R and then expands linearly as shown in the blue curve. The physical wavelength of fluctuation modes is constant in the Hagedorn phase and then increases in proportion to the scale factor $a(t)$ after t_R .

We first compare the ways in which inflation and string gas cosmology, respectively, solve the horizon problem of Standard Big Bang cosmology and lead to the possibility of a causal structure formation mechanism. In inflationary cosmology it is the accelerated expansion of space which renders the horizon much larger than the Hubble radius and ensures that the past light cone of our current observer fits into the horizon at t_{rec} , the time of recombination. In string gas cosmology a long Hagedorn phase will similarly allow the horizon to become much larger than the Hubble radius at t_R . In inflationary cosmology, it is again the accelerated expansion of space which allows fluctuation modes which are currently observed on large scales to be pushed far outside of the Hubble radius. In string gas cosmology, the wavelengths of the perturbation modes is constant during the Hagedorn phase, but the Hubble radius decreases dramatically such that modes become super-Hubble at the end of the Hagedorn phase. In both cases, fluctuations not only have a wavelength smaller than the horizon, but also smaller than the Hubble radius. This enables a causal generation mechanism.

Assuming that the string scale is close to the scale of particle physics

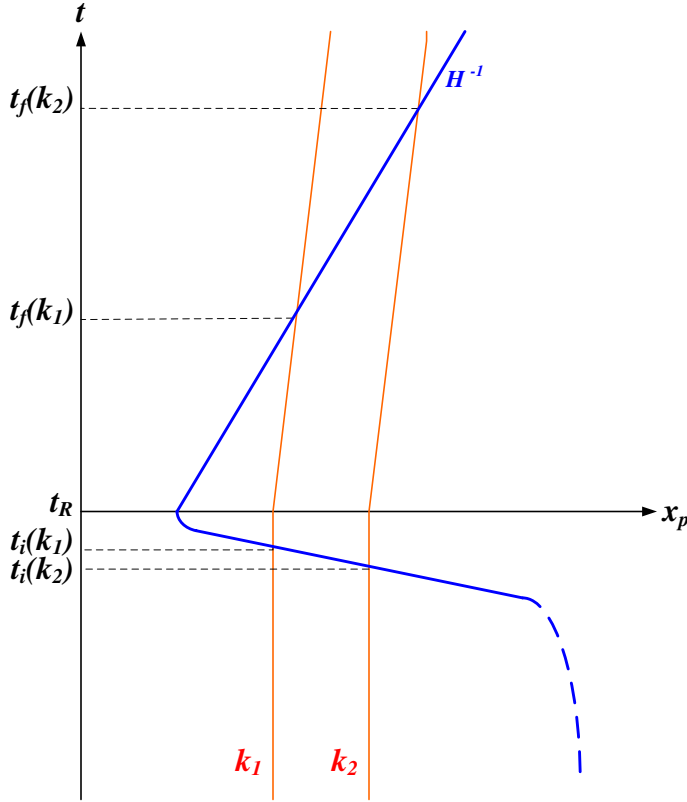


Figure 3. Space-time sketch of string gas cosmology. The vertical axis is time, with t_R denoting the end of the Hagedorn phase. The horizontal axis is physical distance. The blue curve which increases linearly for $t > t_R$ is the Hubble radius which is infinite deep in the Hagedorn phase. The red curves labelled by k_1, k_2 indicate the wavelength of fluctuation modes which have constant comoving length. Since the Hagedorn phase is static, these curves are vertical during this phase. The times $t_i(k)$ and $t_f(k)$ are the instances when the length of the wave k crosses the Hubble radius at the end of the Hagedorn phase and at late times, respectively.

Grand Unification, which is the preferred value for heterotic superstring particle phenomenology [4], the physical wavelength of fluctuation modes which are observed today is of the order 1mm. While this scale seems microscopic from the point of view of cosmology, it is very large compared to the string scale or the Planck scale. In inflationary cosmology, the wavelength of these fluctuations is exponentially smaller than this scale at the beginning of the period of inflation, thus leading to the “trans-Planckian problem” for fluctuations [48]: it is not justified to use Einstein gravity and low energy effective classical matter physics to study the origin and early evolution of

fluctuations. In contrast, in string gas cosmology the fluctuation modes of interest to us are safely in the far infrared for all times, and thus safe from the trans-Planckian problem.

In contrast to the case of inflationary cosmology, in string gas cosmology the inhomogeneities are not vacuum fluctuations, but rather thermal fluctuations. Importantly, they are not thermal fluctuations of a gas of point particles, but of a gas of fundamental strings. Hence, the thermal fluctuations are described by string thermodynamics (see e.g. [17]). The computation of the spectrum of cosmological fluctuations and gravitational waves now proceeds as follows [18, 45]: we first compute the matter fluctuations in the Hagedorn phase, using relations of string thermodynamics [17]. In a second step, we use the Einstein constraint equations to relate the matter fluctuations to metric fluctuations. This is done mode by mode at the time when the wavelength crosses the Hubble radius. Finally, we evolve the metric perturbations on super-Hubble scales until they re-enter the Hubble radius at late times using the equations of the theory of cosmological perturbations (see e.g. [49, 50] for reviews).

Our method relies on three key assumptions: firstly the existence of an initial quasi-static phase containing a thermal gas of strings. Secondly, we posit a fast transition from the Hagedorn phase to the radiation phase of Standard cosmology. Thirdly, we assume the validity of Einstein gravity in the far infrared, an assumption which we use both in converting matter fluctuations to metric ones, and in evolving the perturbations to late times.

We begin with the ansatz for a space-time metric containing both linear cosmological fluctuations (also called “scalar metric fluctuations”) and gravitational waves (“tensor metric fluctuations”):

$$ds^2 = a^2(\eta)((1 + 2\Phi(x, \eta))d\eta^2 - [(1 - 2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j) \quad (1)$$

where η is conformal time, $a(\eta)$ is the scale factor describing the background cosmology, $\Phi(x, \eta)$ are the cosmological perturbations which depend on the spatial coordinates x and on time, and the transverse traceless tensor $h_{ij}(x, \eta)$ describes the gravitational waves (see [49, 50]). We have chosen a gauge (coordinate system) in which the metric corresponding to the cosmological perturbations is diagonal, and assumed that there is no anisotropic stress (which leads to the fact that there is only one non-trivial function Φ characterizing these fluctuations. Note that Latin indices represent spatial coordinates.

The Einstein constraint equations determine the scalar and tensor metric fluctuations in terms of the fluctuations of the energy-momentum tensor. Specifically,

$$\langle |\Phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0_0(k) \delta T^0_0(k) \rangle, \quad (2)$$

and

$$\langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i_j(k) \delta T^i_j(k) \rangle, \quad (3)$$

where G is Newton’s gravitational constant, where in the last equation h is the amplitude of each of the two polarization modes of gravitational waves, and on the right hand side an average of the off-diagonal spatial matrix elements (i.e. $i \neq j$) is implicit.

The expectation values on the right hand side of the above equations indicate thermal expectation values.

Since the wavelengths of the modes we are interested in are always in the far infrared compared to the string scale, we can safely use the perturbed Einstein equations to study the evolution of the fluctuations. From the theory of cosmological fluctuations we know that, since the equation of state parameter $1 + w$ (where w is the ratio of pressure p to energy density ρ) does not change by more than a factor of order one during the transition from the Hagedorn phase to the Standard Cosmology phase (in contrast to what happens in inflationary cosmology during the transition between the inflationary phase and the post-inflation period), both Φ and h remain constant while on super-Hubble scales. Hence, it will be the values of Φ and h computed at Hubble radius crossing at the end of the Hagedorn phase which are relevant for current observations.

Let us now turn to the determination of the initial fluctuations. In general, for thermal fluctuations the energy density perturbations are determined by the specific heat capacity C_V (in a volume V), and by the temperature T . If C_V is the specific heat capacity in a volume of radius R , the resulting energy density variations are given by

$$\langle \delta \rho^2 \rangle = \frac{T^2}{R^6} C_V. \quad (4)$$

What is special about string thermodynamics is that for strings in a compact space of radius R , the specific heat capacity has holographic scaling with R [17, 51]

$$C_V \approx 2 \frac{R^2/l_s^3}{T(1 - T/T_H)}, \quad (5)$$

where l_s is the string length.

By combining these equations we can compute the power spectrum of cosmological fluctuations which is defined as

$$P_\Phi(k) \equiv \frac{1}{2\pi^2} k^3 |\Phi(k)|^2. \quad (6)$$

Inserting the value of $\Phi(k)$ at Hubble radius crossing in terms of the density fluctuations from (2), making use its representation (4) in terms of the specific heat capacity, and then making the replacement (5) we obtain the following expression for the power spectrum in terms of the temperature $T(k)$ when the mode k exits the Hubble radius:

$$P_\Phi(k) = \left(\frac{l_{pl}}{l_s} \right)^4 \frac{T(k)}{T_H} \frac{1}{1 - T(k)/T_H}, \quad (7)$$

where l_{pl} is the Planck length (determined by G).

As can be seen from Figure 1, to first approximation the temperature $T(k)$ is independent of k . To next order, however, we notice that $T(k)$ is a slightly decreasing function of k as k increases, since the temperature starts to decrease as we near the exit from the Hagedorn phase. Thus, string gas cosmology, like inflation, predicts a roughly scale-invariant spectrum of cosmological fluctuations with a slight red tilt [18]. The spectral tilt $n_s - 1$ is given by [52]

$$n_s - 1 = \left(1 - \frac{T(k)}{T_H} \right)^{-1} k \frac{dT(k)/T_H}{dk}, \quad (8)$$

which is negative since $T(k)$ is a decreasing function of k ,

The above computation contains two parameters which from our point of view are free, the first being the ratio of the string length to the Planck length, and the second the deviation of the temperature from T_H during the Hagedorn phase. The latter depends on the total entropy of the system [16], the former is set by the string model. Making use of the value of the string length preferred for particle physics reasons in [4], and taking the ratio of temperatures on the right hand side of (7) to be of the order one, we obtain an amplitude of the spectrum which is consistent with observations. However, from the effective theory analysis which we have presented, we must take l_s and $1 - T(k_0)/T_H$ as free parameters (where k_0 is the pivot scale). These parameters can be fixed by demanding agreement with the current CMB data. Once these parameters are fixed, however, both the amplitude and spectral tilt of the gravitational wave spectrum are determined.

As indicated in (3), the gravitational wave spectrum is given by the off-diagonal pressure fluctuations. String thermodynamics allows the computation of all stress-energy tensor correlation functions. The result for the off-diagonal stress correlation function is [51]

$$\langle |T_{ij}(R)|^2 \rangle \sim \frac{T}{l_s^3 R^4} (1 - T/T_H) \ln^2 \left[\frac{1}{l_s^3 T R^{-2}} (1 - T(k)/T_H) \right]. \quad (9)$$

Note that the factor $(1 - T/T_H)$ is in the numerator instead of in the denominator as in the case of the energy density correlation function. Inserting this expression into (3) yields the following result for the power spectrum of gravitational waves

$$P_h(k) \sim \left(\frac{l_{pl}}{l_s} \right)^4 \frac{T(k)}{T_H} (1 - T(k)/T_H) \ln^2 \left[\frac{1}{l_s^2 k^2} (1 - T(k)/T_H) \right]. \quad (10)$$

This corresponds to a scale-invariant spectrum, but this time with a blue tilt. The tensor tilt n_t is

$$n_t = \frac{1 - 2T(k)}{1 - T(k)/T_H} k \frac{dT(k)/T_H}{dk} = -(n_s - 1) \left(2 \frac{T(k)}{T_H} - 1 \right). \quad (11)$$

To first approximation, the magnitude of the blue tilt of the tensor spectrum equals the magnitude of the red tilt of the scalar spectrum.

The difference in the sign of the tilt of the gravitational wave spectrum is the key and most robust criterion which differentiates between cosmological inflation and string gas cosmology [45, 52]. The reason why the tilt in inflation is negative (i.e. a red spectrum) is that the amplitude of the gravitational wave spectrum is set by the Hubble constant H , and that during inflation $H(t)$ is a decreasing function of time (as long as matter satisfies the “null energy condition”). Thus, long wavelength fluctuations exit the Hubble radius at a larger value of H and thus with a larger amplitude of gravitational waves. On the other hand, in string gas cosmology the gravitational wave spectrum is determined in terms of the off-diagonal pressure fluctuations which are proportional to the pressure. Earlier in the Hagedorn phase the pressure is closer to zero, and hence large-scale modes exit the Hubble radius with a smaller amplitude of the

pressure perturbations than small wavelength modes, and a blue spectrum results. Since the temperature $T(k)$ in the Hagedorn phase is close to T_H , the consistency relations approximately reads

$$n_t \simeq -(n_s - 1). \quad (12)$$

Based on the Planck data [1], string gas cosmology thus predicts

$$n_t = 0.03 \pm 0.01. \quad (13)$$

The tensor to scalar ratio r is also a prediction of string gas cosmology. By combining (7) and (10) we obtain

$$r(k) = (1 - \frac{T(k)}{T_H})^2 \ln^2 \left[\frac{1}{l_s^2 k^2} (1 - \frac{T(k)}{T_H}) \right]. \quad (14)$$

Note that, as in the case of inflationary cosmology, the value of r depends on the scale k .

At this stage, the background of string gas cosmology is not sufficiently developed to be able to make a specific prediction for the tensor to scalar ratio. From (7) and (8) we see that the amplitude and tilt of the scalar spectrum depend on the ratio l_{pl}/l_s , the factor $1 - T/T_H$ and on the $dT(k)/dk$. The last factor is not known since we do not have an analytical description of the exit from the Hagedorn phase. Just considering the factors $(1 - T/T_H)$ we would expect

$$r \sim (1 - T/T_H)^2 \quad (15)$$

which is expected to be significantly smaller than 1. It would be interesting to model the exit from the Hagedorn phase in order to obtain an actual prediction for r .

The Planck and joint Planck/BICEP2 results for r yield a bound of $r < 0.1$. Thus, the results are completely consistent with the predictions of string gas cosmology. To differentiate inflation and string gas cosmology it will be crucial to determine the tensor tilt. The original BICEP2 results [53] favored a blue tilt of the tensor spectrum [54], and hence favored string gas cosmology over inflation as was stressed in [52].

The experimental prospects for measuring the tensor tilt depend on the amplitude r . For a value $r = 0.05$, a careful analysis of B-mode polarization data including de-lensing will allow an identification of a tensor tilt variance of $\sigma(n_t) = 0.04$ [55], very close to the prediction of string gas cosmology. The current upper bound on the tensor tilt are

$$n_t < 0.15 \quad (16)$$

(making use of constraints from pulsar timing, direct detection experiments and nucleosynthesis [56]).

Since the fluctuations in string gas cosmology are of thermal origin, thermal non-Gaussianities will be produced. However, these non-Gaussianities are Poisson suppressed on scales larger than the characteristic scale of the thermal fluctuations, the inverse temperature. Since the temperature in the Hagedorn phase is close to the string scale, the non-Gaussianities on observable scales will be highly suppressed [57].

Hence, observing non-Gaussianities on cosmological scales would be a serious challenge for string gas cosmology. On the other hand, the Planck satellite has not seen any non-Gaussianities, and hence also in this respect the predictions of string gas cosmology are consistent with current observations.

There is one type of non-Gaussianities which could be present in string gas cosmology: if we are dealing with a string theory in which cosmic superstrings [58] are stable (see [59] for a discussion of the criteria for this to be the case), then string gas cosmology would leave behind a network of cosmic strings in our three dimensional space. As studied in detail in the case of cosmic strings (see e.g. [60] for reviews on cosmic strings and cosmology), the network of cosmic superstrings would take on a “scaling solution” in which the network of strings looks identical at all times when all lengths are scaled to the Hubble radius. The scaling solution corresponds to a fixed number N of infinite string segments crossing each Hubble volume, and a distribution of string loops with radius smaller than t which are produced by the interactions between the infinite strings. As a consequence of the energy which is trapped in the strings, cosmic superstrings (like cosmic strings) leave behind clear signatures in cosmological observations. The power spectrum of the string-induced fluctuations is approximately scale-invariant (see e.g. [61]). The non-Gaussianities are prominent in position space maps: line discontinuities in CMB temperature maps [62], rectangles in the sky with direct B-mode polarization [63], and thin wedges in 21cm redshift maps (extended in the sky over degree scale but thin in redshift direction) [64]. The current bound on the string tension μ from not detecting any string-specific signatures is

$$G\mu < 2 \times 10^{-7}. \quad (17)$$

This bound comes from combining Planck data with that of smaller angular scale telescopes [65, 66]. With a dedicated position space search using Planck data, a reduction of this limit might be possible. A study of the potential of the South Pole Telescope to constrain $G\mu$ indicated [67] that an improvement of the bound by one order of magnitude should be possible (for a recent discussion of signals of cosmic strings in new observational windows the reader is referred to [68]).

4. Discussion and Conclusions

Assuming the cosmological background given in Fig. 1, string gas cosmology naturally solves the horizon problem of the Standard Big Bang model. In contrast to inflationary cosmology, the string gas cosmology does not provide a mechanism to produce spatial flatness, and it also assumes a large initial size and entropy of space. As shown above, string gas cosmology leads to a structure formation scenario which yields predictions which are in good agreement with all current observations, and makes predictions for future observations with which the model can be distinguished from cosmological inflation.

The Achilles heel of string gas cosmology is the fact that at the current time we do not have a mathematical description of the Hagedorn phase. In this phase,

non-perturbative string theory will be crucial. Einstein and dilaton gravity are not applicable since these background actions are not consistent with the key symmetries which distinguish string theory from point particle theory. A possible framework to study the dynamics of the Hagedorn phase is “double field theory” [69], a setup in which the number of dimensions is doubled, the two copies being related by T-duality.

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